1 Rational Numbers

Learning Outcomes

After studying this chapter, you will be able to:

- understand properties of rational numbers and use general form of expression to describe them.
- perform basic operations on rational numbers.
- represent rational numbers on the number line.
- understand that between any two rational numbers there lie infinite rational numbers unlike whole numbers.
- make children observe that one can keep on finding more and more rational numbers (as many as one likes) between two given rational numbers.
- solve problems based on real life situations involving rational numbers.

REVISION OF PREVIOUS KNOWLEDGE

Recall what you have learnt in earlier classes.

1. Types of Numbers

Type of Numbers	Natural Numbers (N)	Whole Numbers (W)	Integers (Z or I)	Rational Numbers (Q)
Examples	1, 2, 3, ...	0, 1, 2, 3, ...	..., −3, −2, −1, 0, 1, 2, 3, ...	$-8, -2, -\frac{5}{7}, 0, 2.48, 5\frac{1}{9}, \ldots$

A **rational number** is any number that can be expressed as the quotient of two integers, *i.e.*, in the form $\frac{p}{q}$, where p and q are integers with the condition that $q \neq 0$, *i.e.*, the divisor is **not zero.**

Note!

1. The positive integers are 1, 2, 3, ... The negative integers are −1, −2, −3, ... Zero is neither positive nor negative.
2. Rational numbers include all positive integers, positive fractions, zero, negative integers and negative fractions.
3. *Every fraction is a rational number but a rational number need not be a fraction. A number like* $\frac{7}{-9}$ *is a rational number but not a fraction.*

Positive and Negative Rational Numbers

- A rational number is said to be **positive** if its numerator and denominator are both positive or both negative, e.g., $\frac{4}{7}, \frac{-18}{-35}$.
- A rational number is said to be **negative** if out of its numerator and denominator, one is a positive integer and the other is a negative integer, e.g., $\frac{-9}{14}, \frac{29}{-45}$.

2. Equivalent Rational Numbers

Observe that, $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \ldots = \frac{200}{500} = \ldots$

Numbers like the above which have equal values are called **equivalent rational numbers.**

Rational numbers equivalent to a given rational number can be found by multiplying or dividing as the need may be by the same number.

For example, $\frac{16}{28} = \frac{16\times 4}{28\times 4} = \frac{64}{112}, \frac{16}{28} = \frac{16\div 2}{28\div 2} = \frac{8}{14}$

Standard Form of a Rational Number

A rational number is said to be in its **standard form** or **simplest form** or **lowest terms** if its numerator and the denominator have no common factors.

To express a given rational number in standard form, do as under:

Step 1. Make the denominator positive if not so.

Step 2. Divide both numerator and denominator by their HCF or divide the numerator and the denominator in steps by their common factors normally starting with the least common factor.

Examples:

1. $\frac{16}{-24} = \frac{-16}{24}$ *Make the denominator positive.*

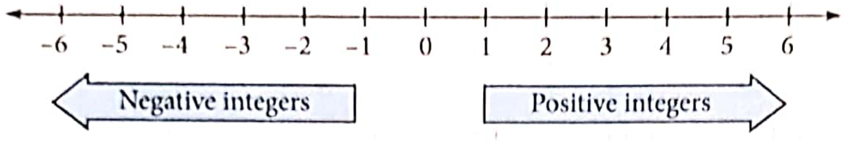
$= \frac{-\cancelto{2}{16}}{\cancelto{3}{24}} = \frac{-2}{3}$ *Divide both the numerator (16) and denominator (24) by their HCF 8.*

Or

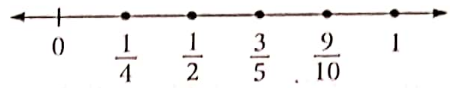
2. $\frac{60}{-84} = \frac{-60}{84} = \frac{-\cancel{60}\,\cancel{30}\,\cancel{15}\,5}{\cancel{84}\,\cancel{42}\,\cancel{21}\,7} = \frac{-5}{7}$ *First, divide both the numerator and denominator by the common factor 2, again by 2 and then by 3.*

3. Rational Numbers on the Number Line

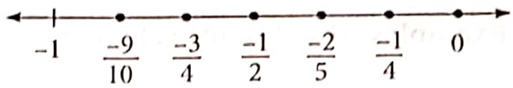
The set of positive integers, 0 and the negative integers can be graphed on a number line as shown below.

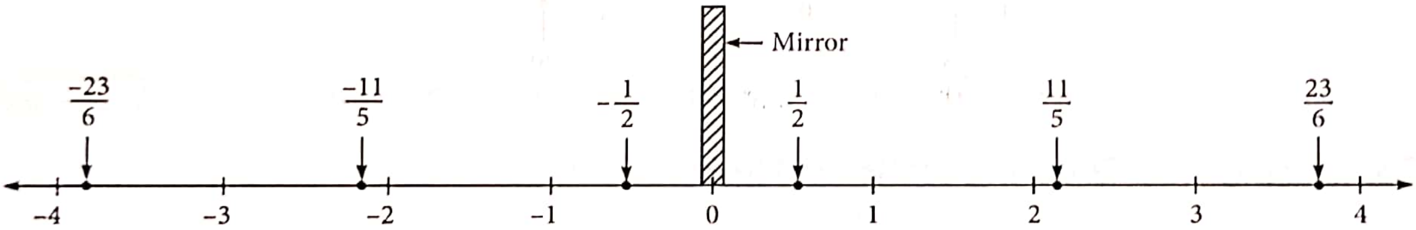


There are gaps between the integers on the number line. These gaps contain fractions. In the gap between 0 and 1, you can write proper fractions such as $\frac{1}{4}, \frac{1}{2}, \frac{3}{5}$ and $\frac{9}{10}$.



The integer 1 is one unit from 0. So, the fraction $\frac{1}{2}$ must be $\frac{1}{2}$ unit from 0. Similarly, we may represent negative rational numbers on the left of 0 on the number line. The rational number $-\frac{1}{2}$ lies halfway between 0 and -1, $-\frac{1}{4}$ is one-fourth of the distance between 0 and -1 and so on.





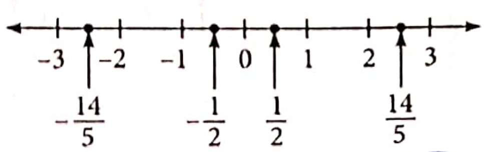
If you imagine that a mirror is placed on the number line at the number 0, then as you look into the mirror, you will see the images of the positive integers. These images are the negative integers. In the same way, fractions such as $\frac{1}{2}$, $\frac{11}{5}$, and $\frac{23}{6}$ each has an opposite in the mirror, *i.e.*, the negative rational numbers $-\frac{1}{2}, \frac{-11}{5}$ and $\frac{-23}{6}$.

Note !

You should understand that unlike with integers, it is not possible to state for definite that so many rational numbers occur between any two given rational numbers. You can always find more and more rational numbers and continue this process indefinitely. No rational number has a unique successor or a unique predecessor.

ABSOLUTE VALUE

Recall that absolute value represents the distance of the number from 0 on the number line and is expressed by writing the number between two vertical bars. Thus, the distance of -3 like that of 3 is also 3 units from 0.



So, $|3| = 3$ and $|-3| = 3$. Similarly, like $\frac{1}{2}$, the number $-\frac{1}{2}$ is also at a distance of $\frac{1}{2}$ units from 0. (Distance is always positive). $\therefore \left|\frac{1}{2}\right| = \frac{1}{2}$ and $\left|-\frac{1}{2}\right| = \frac{1}{2}$

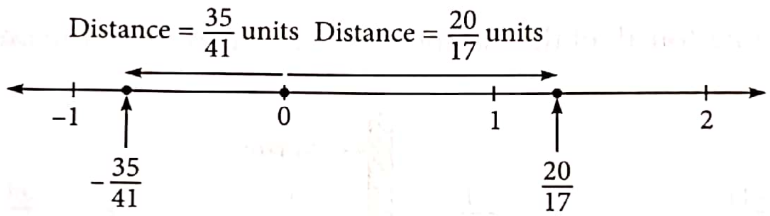
The absolute value of a rational number is a rational number without any regard to its sign.

Note!

The absolute value of 0 is 0, *i.e.*, $|0| = 0$.

Examples: The absolute values of $\frac{-35}{41}$ and $\frac{20}{17}$ are $\left|\frac{-35}{41}\right| = \frac{35}{41}$ and $\left|\frac{20}{17}\right| = \frac{20}{17}$.

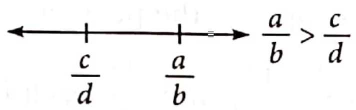
They are shown on the number line below.



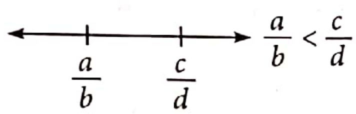
Comparing and Ordering Rational Numbers

Method 1. *By using a number line*

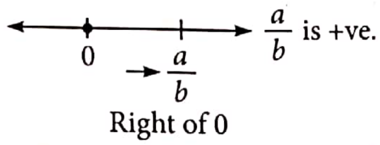
- If $\frac{a}{b}$ lies to the right of $\frac{c}{d}$, then $\frac{a}{b} > \frac{c}{d}$.



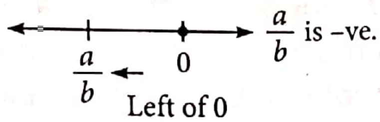
- If $\frac{a}{b}$ lies to the left of $\frac{c}{d}$, then $\frac{a}{b} < \frac{c}{d}$.



- All rational numbers lying to the right of 0 are **positive**.

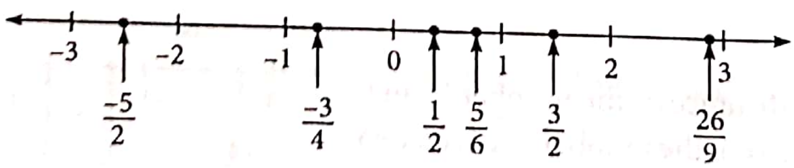


- All rational numbers lying to the left of 0 are **negative**.



- A positive rational number is always greater than a negative rational number.

Example:



(i) Since $\frac{5}{6}$ is located on the right of $\frac{1}{2}$, so $\frac{5}{6} > \frac{1}{2}$. Similarly, $\frac{26}{9} > \frac{3}{2}, \frac{-3}{4} > -1, -2 > \frac{-5}{2}, 2 > \frac{-3}{4}$ and so on.

(ii) Since $\frac{5}{6}$ is located on the left of 1, so $\frac{5}{6} < 1$. Similarly, $\frac{3}{2} < \frac{26}{9}, -1 < -\frac{3}{4}, -3 < \frac{-5}{2}$ and so on.

Method 2. *It is not always easy and practical to compare two rational numbers by using a number line. You can compare two given rational numbers* ***by making their denominators same and then compare the numerators.***

Example 1 **Compare $\frac{5}{14}$ and $\frac{9}{21}$.**

Solution LCM of 14 and 21 = 7 × 2 × 3 = 42

$$\begin{array}{r|l} 7 & 14, 21 \\ \hline & 2,\ 3 \end{array}$$

So, $\frac{5}{14} = \frac{5 \times 3}{14 \times 3} = \frac{15}{42}, \frac{9}{21} = \frac{9 \times 2}{21 \times 2} = \frac{18}{42}$

Make denominators same.
Here, denominators = their LCM, i.e., 42.

Since 18 > 15, so $\frac{18}{42} > \frac{15}{42}$, *i.e.,* $\frac{9}{21} > \frac{5}{14}$.

Compare numerators

Example 2 **Compare $\frac{-7}{18}$ and $\frac{23}{-24}$.**

Solution Making denominators positive, the given rational numbers are $\frac{-7}{18}$ and $\frac{-23}{24}$.

Express $\frac{23}{-24}$ in standard form.

LCM of 18 and 24 = 72

$$\begin{array}{r|l} 6 & 18, 24 \\ \hline & 3,\ 4 \end{array}$$

Make denominators same.

$\therefore \frac{-7}{18} = \frac{-7 \times 4}{18 \times 4} = \frac{-28}{72}, \frac{-23}{24} = \frac{-23 \times 3}{24 \times 3} = \frac{-69}{72}$

Compare numerators.

Since −28 > −69, so $\frac{-28}{72} > \frac{-69}{72}$, *i.e.,* $\frac{-7}{18} > \frac{23}{-24}$.

Method 3. ***By changing into decimals***

Example 3 **Compare $\frac{7}{8}$ and $\frac{3}{20}$.**

Solution $\frac{7}{8} = 0.875, \frac{3}{20} = 0.15$. Since 0.875 > 0.15 so, $\frac{7}{8} > \frac{3}{20}$.

Method 4. ***By using cross product rule***

If $ad > bc$, then $\frac{a}{b} > \frac{c}{d}$

$$\frac{a}{b} \times \frac{c}{d} \qquad a \times d \quad b \times c$$

If $ad < bc$, then $\frac{a}{b} < \frac{c}{d}$.

Maths Alert!
Before applying cross product rule, make the denominators positive if not so.

Example 4 Compare $-\dfrac{17}{15}$ and $\dfrac{8}{-9}$.

Solution The given rational numbers are $\dfrac{-17}{15}$ and $\dfrac{-8}{9}$.

1. Make denominators positive.

2. Place the –ve sign with the numerator.

$- 17 \times 9 = - 153$

$15 \times -8 = - 120$

$\dfrac{-17}{15} \times \dfrac{-8}{9}$

Since $-153 < - 120$, so $-\dfrac{17}{15} < \dfrac{-8}{9}$, *i.e.*, $-\dfrac{17}{15} < \dfrac{8}{-9}$.

Self Practice 1A

1. **Write four rational numbers equivalent to each of the following rational numbers**

   (a) $\dfrac{2}{9}$ (b) $\dfrac{-3}{5}$ (c) $\dfrac{7}{-11}$ (d) $\dfrac{-6}{-13}$

2. **Write the following rational numbers in standard form.**

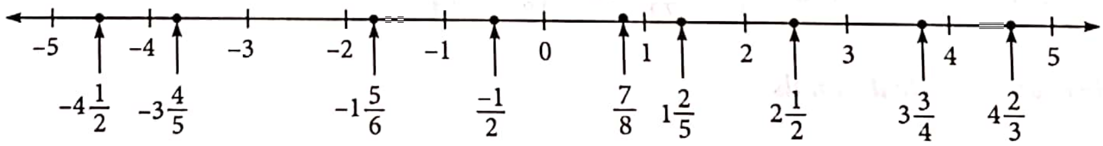
   $\dfrac{15}{20}, \dfrac{64}{72}, \dfrac{-54}{-63}, \dfrac{88}{-99}, -\dfrac{87}{156}$

3. **Write the absolute value of each of the following.**

   $\dfrac{17}{24}, \dfrac{-6}{35}, \dfrac{-11}{-17}, 0, \dfrac{28}{-45}$

4. Write the rational numbers whose absolute value is $\dfrac{5}{6}$.

5. **Using the number line shown below fill in the blanks by writing '>' or '<'.**



   (a) -1 ____ $-\dfrac{1}{2}$ (b) 1 ____ -2 (c) 0 ____ -3 (d) -5 ____ -4

   (e) $3\dfrac{3}{4}$ ____ $1\dfrac{2}{5}$ (f) -4 ____ $4\dfrac{2}{3}$ (g) $-3\dfrac{4}{5}$ ____ -4 (h) $-\dfrac{1}{2}$ ____ $-1\dfrac{5}{6}$

6. **Which pairs of rational numbers are not equivalent?**

   (a) $\dfrac{15}{19}, \dfrac{-15}{-19}$ (b) $\dfrac{28}{-49}, \dfrac{-4}{7}$ (c) $\dfrac{16}{17}, \dfrac{26}{27}$ (d) $\dfrac{27}{99}, \dfrac{51}{187}$ (e) $\dfrac{16}{20}, \dfrac{4}{-5}$

7. **Compare. Write <, > or = in the blanks to make a true statement.**

   (a) $-\dfrac{8}{15}$ ____ $-\dfrac{7}{30}$ (b) $-\dfrac{1}{5}$ ____ $-\dfrac{1}{4}$ (c) $\dfrac{3}{10}$ ____ $\dfrac{5}{12}$ (d) $\dfrac{5}{6}$ ____ $\dfrac{15}{18}$

(e) $\frac{1}{-25}$ ____ $\frac{1}{50}$ (f) $\frac{17}{40}$ ____ 0.519 (g) -8.9 ____ $-8\frac{1}{4}$ (h) $\frac{-1}{8}$ ____ $\frac{4}{-32}$

(i) $\frac{18}{-19}$ ____ $\frac{-10}{11}$ (j) $-\frac{16}{25}$ ____ $\frac{17}{-21}$ (k) $\frac{-5}{3}$ ____ $\frac{40}{27}$ (l) $\frac{15}{-16}$ ____ $-\frac{9}{10}$

8. **Arrange the following groups of rational numbers in ascending order.**
[**Hint.** It is easier to do so by converting to decimals.]

(a) $-\frac{1}{4}, \frac{1}{2}, \frac{7}{8}, \frac{15}{16}, -\frac{9}{40}$ (b) $\frac{5}{7}, \frac{-7}{25}, \frac{9}{-4}, \frac{-6}{5}, \frac{8}{11}$

9. **Arrange the following groups of rational numbers in descending order.**

(a) $-1\frac{5}{8}, 1\frac{11}{12}, -1\frac{11}{16}, \frac{17}{9}, -1\frac{5}{6}$ (b) $\frac{-7}{16}, \frac{-1}{4}, \frac{1}{8}, \frac{9}{32}, \frac{18}{25}$

OPERATIONS ON RATIONAL NUMBERS

A. Adding and Subtracting Rational Numbers

Case 1. Adding and subtracting with like denominators.

Rule *To add or subtract rational numbers with the same denominator, add or subtract the numerators and keep the denominator same.*

$$\frac{8}{19}+\frac{5}{19}=\frac{8+5}{19}=\frac{13}{19} \qquad \frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}\ (c \neq 0)$$

$$\frac{7}{13}-\frac{2}{13}=\frac{7-2}{13}=\frac{5}{13} \qquad \frac{a}{c}-\frac{b}{c}=\frac{a-b}{c}\ (c \neq 0)$$

While adding or subtracting negative rational numbers or mixed numbers, keep in mind that you can write

$-\frac{11}{15}=\frac{-11}{15}$ or $\frac{11}{-15}$ and $-6\frac{7}{9}=-6-\frac{7}{9}$ or $-6+\frac{-7}{9}$

Examples: 1. $-\frac{2}{7}+\frac{3}{7}=\frac{-2}{7}+\frac{3}{7}=\frac{-2+3}{7}=\frac{1}{7}$

2. $\frac{8}{21}-\frac{4}{21}=\frac{8}{21}+\frac{-4}{21}=\frac{8+(-4)}{21}=\frac{4}{21}$

3. $-\frac{3}{8}-\frac{1}{8}=\frac{-3}{8}+\frac{-1}{8}=\frac{-3+(-1)}{8}=\frac{-4}{8}=-\frac{1}{2}$.

4. $-6\frac{5}{7}+2\frac{3}{7}=-6-\frac{5}{7}+2+\frac{3}{7}=(-6+2)-\frac{5}{7}+\frac{3}{7}=-4-\frac{2}{7}=-4\frac{2}{7}$.

Second Method. *By rewriting the given rational numbers as improper fractions.*

$$-6\frac{5}{7}+2\frac{3}{7}=\frac{-47}{7}+\frac{17}{7}=\frac{-47+17}{7}=\frac{-30}{7}=-\frac{30}{7}=-4\frac{2}{7}.$$

5. $\frac{3}{19}-\frac{4}{19}+\frac{8}{19}=\frac{3-4+8}{19}$ *Write 3 – 4 + 8 over common denominator.*

$=\frac{7}{19}$ *Evaluate numerator from left to right.*

6. $4\frac{5}{11}-3\frac{2}{11}-9\frac{8}{11}=(4-3-9)+\left(\frac{5}{11}-\frac{2}{11}-\frac{8}{11}\right)$ *Group whole numbers and fractions.*

$=-8+\frac{5-2-8}{11}$ *Evaluate inside parentheses.*

$=-8-\frac{5}{11}=-8\frac{5}{11}.$

Second Method. $4\frac{5}{11}-3\frac{2}{11}-9\frac{8}{11}=\frac{49}{11}-\frac{35}{11}-\frac{107}{11}$ *Write as improper fractions.*

$=\frac{49-35-107}{11}=\frac{-93}{11}=-8\frac{5}{11}.$

Case 2. Adding and subtracting with different denominators

Method. Step 1. *Find least common denominator (LCD). This is equal to the LCM of the denominators of the given rational numbers.*

Step 2. *Then work out as shown in the following examples. This is the same as you do with fractions.*

Examples: 1. $\frac{5}{-7}+\frac{3}{4}=\frac{-5}{7}+\frac{3}{4}$ Think LCD = LCM of 7 and 4 = 7 × 4 = 28

$=\frac{4\times(-5)+7\times3}{28}$

$=\frac{-20+21}{28}=\frac{1}{28}.$ *28 ÷ 7 = 4. Write 4 × – 5 = –20*
28 ÷ 4 = 7. Write 7 × 3 = 21

2. $3\frac{1}{8}-5\frac{1}{12}=\frac{25}{8}-\frac{61}{12}$ *Write as improper fractions.*

$=\frac{3\times25-2\times61}{24}$ *LCD = LCM of 8 and 12 = 24*

$=\frac{75-122}{24}=\frac{-47}{24}=-1\frac{23}{24}$

3. $5\frac{1}{12}+7\frac{1}{18}-1\frac{1}{36}=(5+7-1)+\frac{1}{12}+\frac{1}{18}-\frac{1}{36}=11+\frac{6+4-2}{72}=11+\frac{8}{72}=11+\frac{1}{9}=11\frac{1}{9}.$

Note !
You may also do this sum by writing the given rational numbers as improper fractions as done in example 2.

Self Practice 1B

Find the sum or difference. Simplify if possible.

1. (a) $\frac{7}{20}+\frac{11}{20}$ (b) $\frac{1}{12}+\frac{7}{12}$ (c) $\frac{3}{10}-\frac{7}{10}$ (d) $-5\frac{2}{7}-3\frac{3}{7}$

   (e) $\frac{112}{13}+\frac{-229}{13}$ (f) $\frac{-243}{58}+\frac{719}{58}+\frac{273}{-58}$ (g) $-8\frac{4}{5}-\frac{3}{5}$

2. (a) $\frac{5}{7}+\frac{-3}{8}$ (b) $\frac{7}{18}-\frac{5}{6}$ (c) $\frac{3}{8}-\frac{5}{32}$ (d) $\frac{7}{-9}+\frac{1}{-6}$

   (e) $5\frac{4}{5}+\left(-3\frac{2}{7}\right)$ (f) $9-11\frac{4}{7}$ (g) $-8\frac{4}{11}-(-9)$

Simplify:

3. $\frac{3}{4}-\frac{5}{7}+\frac{9}{14}$ 4. $5\frac{7}{9}-1\frac{1}{10}-7\frac{11}{15}$

B. Multiplying Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ $(b, d \neq 0)$ are two rational numbers, then $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}=\frac{\text{Product of numerators}}{\text{Product of denominators}}$

Also, keep in mind that

$(+)\,.\,(+) = (+)$, or $(-)\,.\,(-) = (+)$

$(+)\,.\,(-) = (-)$, or $(-)\,.\,(+) = (-)$

i.e., product of same signs is positive and product of different signs is negative.

Examples: 1. $\frac{5}{7}\times\frac{3}{11}=\frac{5\times3}{7\times11}=\frac{15}{77}$.

2. $-\frac{3}{5}\cdot\left(-\frac{4}{7}\right)=\frac{-3\times-4}{5\times7}=\frac{12}{35}$.

3. $-2\frac{4}{9}\times1\frac{4}{5}=-\frac{22}{9}\times\frac{9}{5}=\frac{-22\times\overset{1}{\cancel{9}}}{\underset{1}{\cancel{9}}\times5}=\frac{-22}{5}$.

4. $20\times\left(-1\frac{7}{10}\right)=20\times\left(\frac{-17}{10}\right)=\frac{\overset{2}{\cancel{20}}\times(-17)}{\underset{1}{\cancel{10}}}=-34$.

C. Dividing Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ $(b, d \neq 0)$ are two rational numbers, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$, *i.e.*, to divide by a rational number, multiply by its **reciprocal**.

Reciprocal

Also, recall that $(+) \div (+) = +$; $(-) \div (-) = +$

$(+) \div (-) = -$; $(-) \div (+) = -$

i.e., if both rational numbers are of the same sign, the quotient is positive and if both rational numbers are of opposite sign, the quotient is negative.

Examples: 1. $\frac{5}{14} \div \frac{7}{10} = \frac{5}{\cancel{14}_{7}} \times \frac{\cancel{10}^{5}}{7}$ *Multiply by the reciprocal of* $\frac{7}{10}$.

$= \frac{5 \times 5}{7 \times 7} = \frac{25}{49}$

2. $\frac{21}{32} \div 7 = \frac{\cancel{21}^{3}}{32} \times \frac{1}{\cancel{7}_{1}} = \frac{3}{32}$ *Multiply by the reciprocal of 7, i.e.,* $\frac{1}{7}$.

3. $-15\frac{3}{4} \div \left(-2\frac{5}{8}\right) = \frac{-63}{4} \div -\frac{21}{8}$ *Write as improper fractions.*

$= \frac{-\cancel{63}^{\cancel{9}^{3}}}{\cancel{4}_{1}} \times \frac{-\cancel{8}^{2}}{\cancel{21}_{\cancel{3}_{1}}}$ *Multiply by the reciprocal of* $-\frac{21}{8}$.

$= (-3) \times (-2) = 6.$

Self Practice 1C

Multiply:

1. (a) $-\frac{3}{4} \times \left(-\frac{8}{9}\right)$ (b) $\frac{7}{20} \times \left(-\frac{5}{14}\right)$ (c) $-\frac{24}{49} \times \frac{35}{36}$ (d) $\frac{17}{30} \times \frac{45}{51}$

2. (a) $6 \times 1\frac{2}{3}$ (b) $-18 \times \left(-4\frac{5}{9}\right)$ (c) $2\frac{1}{9} \times \frac{-7}{38}$

3. $\frac{25}{4} \times \left(\frac{-8}{9}\right) \times \left(-3\frac{9}{11}\right) \times \left(\frac{22}{-35}\right)$

Divide:

4. (a) $\frac{44}{65} \div \frac{11}{13}$ (b) $-\frac{8}{17} \div \frac{24}{-51}$ (c) $\frac{-2}{25} \div \frac{4}{35}$ (d) $115 \div \frac{-23}{5}$

5. (a) $-1\frac{11}{14} \div \left(-1\frac{7}{18}\right)$ (b) $6\frac{1}{3} \div \left(-2\frac{5}{6}\right)$ (c) $-5\frac{1}{4} \div 7$ (d) $1\frac{1}{12} \div 2\frac{5}{17}$

Simplify:

6. $\left(\frac{4}{9} \div \frac{4}{7}\right) \times 1\frac{2}{7}$ 7. $\left[5\frac{1}{2} \times \left(-1\frac{5}{6}\right)\right] \div 2\frac{3}{4}$ 8. $\left(-\frac{8}{5} \div 12\right) \times \left(\frac{-5}{6} \div (-2)\right)$

PROPERTIES OF OPERATIONS ON RATIONAL NUMBERS

The operations of addition and multiplication obey all the laws in rational numbers (Q) that they obey in integers (I). In addition, they have the inverse property of multiplication in Q which they don't possess in I. Let a, b, c be any rational numbers.

Property 1. Closure Property

Addition. ***Rational numbers are closed under addition,*** that is, $a + b$ is a unique rational number.

Examples: $\frac{3}{5} + \frac{1}{4} = \frac{17}{20}$, which is a rational number.

$\frac{-8}{9} + \frac{5}{18} = \frac{-11}{18}$, which is a rational number.

Subtraction. ***Rational numbers are closed under subtraction,*** that is, $a - b$ is a unique rational number.

Example: $\frac{6}{7} - \frac{3}{14} = \frac{12-3}{14} = \frac{9}{14}$, which is a rational number.

Multiplication. ***Rational numbers are closed under multiplication,*** that is, $a \,.\, b$ is a rational number.

Example: $\frac{5}{-9} \cdot \frac{18}{25} = -\frac{\cancel{5}^{1}}{\cancel{9}_{1}} \cdot \frac{\cancel{18}^{2}}{\cancel{25}_{5}} = -\frac{2}{5}$, which is a rational number.

Division. ***Rational numbers are not closed under division.***

$-\frac{7}{16} \div \frac{21}{32} = \frac{-\cancel{7}^{1}}{\cancel{16}_{1}} \times \frac{\cancel{32}^{2}}{\cancel{21}_{3}} = \frac{-2}{3}$, which is a rational number. But if $b = 0$, then $a \div 0$ is not defined.

For example, $\frac{7}{11} \div 0$ is not defined, so rational numbers are NOT closed under division.

Property 2. Commutativity Property

Addition. ***Addition is commutative for rational numbers,*** that is, $a + b = b + a$.

Example: $-\dfrac{4}{9}+\dfrac{1}{18}=\dfrac{-8+1}{18}=\dfrac{-7}{18};\ \dfrac{1}{18}+\dfrac{-4}{9}=\dfrac{1+(-8)}{18}=\dfrac{-7}{18}$

$\therefore \quad \left(-\dfrac{4}{9}\right)+\dfrac{1}{18}=\dfrac{1}{18}+\left(\dfrac{-4}{9}\right)$

Subtraction. ***Subtraction is not commutative for rational numbers,*** that is, $a - b \neq b - a$.

Example: $\dfrac{3}{4}-\dfrac{2}{5}=\dfrac{15-8}{20}=\dfrac{7}{20};\ \dfrac{2}{5}-\dfrac{3}{4}=\dfrac{8-15}{20}=\dfrac{-7}{20}$

$\therefore \quad \dfrac{3}{4}-\dfrac{2}{5}\neq\dfrac{2}{5}-\dfrac{3}{4}$ $\Rightarrow$ Subtraction is not commutative.

Multiplication. ***Multiplication is commutative for rational numbers,*** that is, $a.b = b.a$.

Example: $\dfrac{7}{8}\times\dfrac{-5}{11}=\dfrac{-35}{88};\ \dfrac{-5}{11}\times\dfrac{7}{8}=\dfrac{-35}{88}$

$\therefore \quad \dfrac{7}{8}\times\dfrac{-5}{11}=\dfrac{-5}{11}\times\dfrac{7}{8}$ $\Rightarrow$ Multiplication is commutative.

Division. ***Division is not commutative for rational numbers,*** that is, $a \div b \neq b \div a$.

Example: $\dfrac{-5}{11}\div\left(\dfrac{15}{-22}\right)=\dfrac{-\cancel{5}^{1}}{\cancel{11}_{1}}\times\dfrac{-\cancel{22}^{2}}{\cancel{15}_{3}}=\dfrac{-1\times-2}{1\times3}=\dfrac{2}{3}$

$\dfrac{15}{-22}\div\left(\dfrac{-5}{11}\right)=-\dfrac{\cancel{15}^{3}}{\cancel{22}_{2}}\times\dfrac{-\cancel{11}^{1}}{\cancel{5}_{1}}=\dfrac{-3\times-1}{2\times1}=\dfrac{3}{2}$

$\therefore \quad \dfrac{-5}{11}\div\left(\dfrac{15}{-22}\right)\neq\dfrac{15}{-22}\div\left(\dfrac{-5}{11}\right)$ $\Rightarrow$ Division is not commutative.

Property 3. Associativity Property

Addition. ***Addition is associative for rational numbers,*** that is, $(a + b) + c = a + (b + c)$.

Example: $\left(\dfrac{-2}{3}+\dfrac{1}{6}\right)+\left(\dfrac{-7}{12}\right)=\left(\dfrac{-4+1}{6}\right)+\left(\dfrac{-7}{12}\right)=\dfrac{-1}{2}+\left(\dfrac{-7}{12}\right)=\dfrac{-13}{12}.$

$\dfrac{-2}{3}+\left[\dfrac{1}{6}+\left(\dfrac{-7}{12}\right)\right]=\dfrac{-2}{3}+\dfrac{-5}{12}=\dfrac{-13}{12}$

$\therefore \quad \left(\dfrac{-2}{3}+\dfrac{1}{6}\right)+\left(\dfrac{-7}{12}\right)=\dfrac{-2}{3}+\left[\dfrac{1}{6}+\left(\dfrac{-7}{12}\right)\right]$

$\Rightarrow$ Addition is associative for rational numbers.

Subtraction. ***Subtraction is not associative for rational numbers,*** that is, $(a - b) - c \neq a - (b - c)$.

Example: $\left[\frac{-3}{4} - \left(\frac{-5}{6}\right)\right] - \frac{1}{3} = \left(\frac{-9+10}{12}\right) - \frac{1}{3} = \frac{1}{12} - \frac{1}{3} = \frac{1-4}{12} = \frac{-3}{12} = \frac{-1}{4}.$

$$\frac{-3}{4} - \left(\frac{-5}{6} - \frac{1}{3}\right) = \frac{-3}{4} - \left(\frac{-7}{6}\right) = \frac{-9+14}{12} = \frac{5}{12}$$

Thus, $\left[\frac{-3}{4} - \left(\frac{-5}{6}\right)\right] - \frac{1}{3} \neq \frac{-3}{4} - \left(\frac{-5}{6} - \frac{1}{3}\right)$

So, subtraction is not associative for rational numbers.

Multiplication. ***Multiplication is associative for rational numbers,*** that is, $(a \times b) \times c = a \times (b \times c)$.

Example: $\left[\frac{7}{8} \times \left(\frac{-16}{21}\right)\right] \times \left(\frac{-3}{16}\right) = \frac{-2}{3} \times \frac{-3}{16} = \frac{1}{8}$

$$\frac{7}{8} \times \left[\left(\frac{-16}{21}\right) \times \left(\frac{-3}{16}\right)\right] = \frac{7}{8} \times \frac{1}{7} = \frac{1}{8}$$

$\therefore$ $\left[\frac{7}{8} \times \left(\frac{-16}{21}\right)\right] \times \left(\frac{-3}{16}\right) = \frac{7}{8} \times \left[\left(\frac{-16}{21}\right) \times \left(\frac{-3}{16}\right)\right]$

$\Rightarrow$ Multiplication is associative for rational numbers.

Division. ***Division is not associative for rational numbers,*** that is, $(a \div b) \div c \neq a \div (b \div c)$.

Example: $\left[\frac{5}{12} \div \left(\frac{-15}{16}\right)\right] \div \left(\frac{-10}{27}\right) = \frac{-4}{9} \times \left(\frac{-27}{10}\right) = 1\frac{1}{5}$

$$\frac{5}{12} \div \left[\left(\frac{-15}{16}\right) \div \left(\frac{-10}{27}\right)\right] = \frac{5}{12} \div \frac{81}{32} = \frac{5}{12} \times \frac{32}{81} = \frac{40}{243}$$

Thus, $\left[\frac{5}{12} \div \left(\frac{-15}{16}\right)\right] \div \left(\frac{-10}{27}\right) \neq \frac{5}{12} \div \left[\left(\frac{-15}{16}\right) \div \left(\frac{-10}{27}\right)\right]$

$\Rightarrow$ Division is not associative for rational numbers.

Property 4. Distributivity Property

Distributivity of multiplication over addition and subtraction.

Multiplication is distributive over addition and subtraction,

that is $a(b + c) = ab + ac$; $a(b - c) = ab - ac$

Example: Let $a = \frac{-1}{2}$, $b = \frac{-3}{5}$, $c = \frac{2}{7}$, then

$$a(b + c) = \left(\frac{-1}{2}\right) \cdot \left[\left(\frac{-3}{5}\right) + \frac{2}{7}\right] = \frac{-1}{2} \times \frac{-11}{35} = \frac{11}{70}.$$

$$ab = \frac{-1}{2}\times\left(\frac{-3}{5}\right) = \frac{3}{10},\ ac = \left(\frac{-1}{2}\right)\times\frac{2}{7} = \frac{-1}{7}$$

$$\therefore \quad ab + ac = \frac{3}{10}+\left(\frac{-1}{7}\right) = \frac{21-10}{70} = \frac{11}{70}.$$

Thus, $a(b + c) = ab + ac \Rightarrow$ Multiplication is distributive over addition.

Also, $a(b - c) = \left(\frac{-1}{2}\right)\cdot\left[\left(\frac{-3}{5}\right)-\frac{2}{7}\right] = \frac{-1}{2}\times\frac{-31}{35} = \frac{31}{70}$

$$ab - ac = \frac{3}{10}-\left(\frac{-1}{7}\right) = \frac{3}{10}+\frac{1}{7} = \frac{31}{70}.$$

Thus, $a(b - c) = ab - ac \Rightarrow$ Multiplication is distributive over subtraction.

Property 5. Property of Zero (Additive Identity)

The sum of any rational number a and 0 is the rational number itself, that is, for any rational number 'a', $a + 0 = 0 + a = a$.

Example: $\frac{1}{2}+0=\frac{1}{2}, \frac{-5}{17}+0=\frac{-5}{17}$.

Since the identity of the number remains unchanged, number '0' is called the **additive identity**.

Property 6. Property of One (Multiplicative Identity)

The product of any rational number and 1 is the rational number itself, that is, for any rational number 'a', $a \times 1 = 1 \times a = a$.

Example: $7\times1=7, \frac{6}{11}\times1=\frac{6}{11}, \frac{-2}{5}\times1=\frac{-2}{5}$

The number '1' is called the **multiplicative identity.**

Property 7. Additive Inverse (Negative of a Number)

For any rational number a, $a + (-a) = (-a) + a = 0$. The rational number '$-a$' is the additive inverse of 'a' because it makes the sum of the two, equal to zero and vice versa.

Example: $\left(\frac{-11}{18}\right)+\frac{11}{18}=\frac{11}{18}+\left(\frac{-11}{18}\right)=0$

So, $\frac{-11}{18}$ is the additive inverse of $\frac{11}{18}$ and $\frac{11}{18}$ is the additive inverse of $\frac{-11}{18}$.

Property 8. Multiplicative Inverse or Reciprocal of a Number

If a is any non-zero rational number and $a\cdot\frac{1}{a}=\frac{1}{a}\cdot a=1$, then $\frac{1}{a}$ is called the reciprocal or multiplicative inverse of a.

Example: The reciprocal of 9 is $\frac{1}{9}$ since $9 \times \frac{1}{9} = 1$. The reciprocal of -1 is -1. The reciprocal of -8 is $-\frac{1}{8}$ and the reciprocal of $\frac{-7}{15}$ is $\frac{-15}{7}$.

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Self Practice 1D

1. State the property used by the following statements.

(a) $\frac{-7}{8}\times\frac{11}{15}=\frac{11}{15}\times\frac{-7}{8}$ (b) $\frac{6}{7}\times\frac{7}{6}=1$ (c) $-\frac{1}{8}+\frac{1}{8}=0$

(d) $\frac{-21}{29}+\frac{6}{19}=\frac{6}{19}+\left(\frac{-21}{29}\right)$ (e) $\frac{-23}{70}\times 1=\frac{-23}{70}$ (f) $\frac{5}{8}\left(\frac{1}{2}-\frac{1}{3}\right)=\frac{5}{8}\times\frac{1}{2}-\frac{5}{8}\times\frac{1}{3}$

(g) $\left(\frac{1}{2}\times\frac{1}{3}\right)\times\frac{1}{5}=\frac{1}{2}\times\left(\frac{1}{3}\times\frac{1}{5}\right)$ (h) $\left(\frac{1}{2}\times\frac{1}{3}\right)\times\frac{1}{5}=\frac{1}{2}\times\left(\frac{1}{3}\times\frac{1}{5}\right)$

2. Verify the following and state the property used.

(a) $\frac{17}{135}\times\frac{15}{-51}=\frac{15}{-51}\times\frac{17}{135}$ (b) $\frac{1}{9}\left(\frac{18}{5}+\frac{-3}{20}\right)=\frac{1}{9}\times\frac{18}{5}+\frac{1}{9}\times\left(\frac{-3}{20}\right)$

(c) $\left(-\frac{1}{2}+\frac{3}{7}\right)+\left(\frac{-4}{3}\right)=-\frac{1}{2}+\left[\frac{3}{7}+\left(\frac{-4}{3}\right)\right]$ (d) $\left(\frac{5}{3}\times\frac{-4}{5}\right)\times\frac{3}{5}=\frac{5}{3}\times\left[\left(\frac{-4}{5}\right)\times\frac{3}{5}\right]$

(e) $\frac{-19}{20}\times 1=1\times\left(\frac{-19}{20}\right)=\frac{-19}{20}$ (f) $\frac{-17}{24}\times\frac{24}{-17}=1$

(g) $\frac{-2}{3}+0=0+\left(\frac{-2}{3}\right)=\frac{-2}{3}$ (h) $\frac{1}{7}+0=0+\frac{1}{7}=\frac{1}{7}$

3. Taking, $a=\frac{4}{7}$, $b=\frac{-5}{2}$ and $c=\frac{4}{3}$, show that

$a\div(b\div c)\neq(a\div b)\div c$, that is, division is not associative for rational numbers.

4. Using distributivity, find $\left(\frac{8}{15}\times\left(\frac{-7}{18}\right)\right)+\left(\frac{8}{15}\times\frac{-11}{18}\right)$.

5. Find the additive inverse of each of the following.

(a) $\frac{5}{16}$ (b) $\frac{-15}{-16}$ (c) $\frac{-8}{19}$ (d) $\frac{20}{-23}$

6. Write the multiplicative inverse of each of the following.

(a) -7 (b) 10 (c) $\frac{17}{41}$ (d) $\frac{28}{-59}$

(e) $\frac{-29}{-36}$ (f) $\left(\frac{1}{3}-\frac{1}{4}\right)\times(-2)$ (g) $\frac{5}{8}\div\frac{15}{16}\times\left(\frac{-3}{2}\right)$ (h) $16\div(-32)$

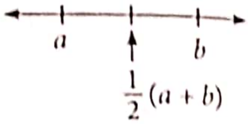
INSERTION OF ONE OR MORE RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS

Denseness Property of Rational Numbers

You have already seen that there are infinitely many rational numbers between any two different rational numbers.

Case 1. To find one rational number between two given rational numbers

Let a and b be two rational numbers such that $b > a$. Then, $\frac{a+b}{2}$ is a rational number lying between a and b.



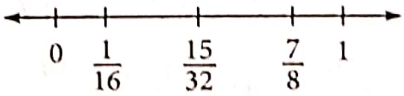
Example 5 **Find a rational number between**

(a) $\frac{7}{8}$ and $\frac{1}{16}$ (b) $\frac{-4}{5}$ and $\frac{7}{10}$

Solution (a) We have, $\frac{1}{16} < \frac{7}{8}$.

One rational number between $\frac{1}{16}$ and $\frac{7}{8}$ is

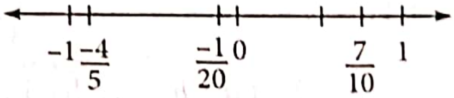
$$\frac{1}{2}\left(\frac{1}{16}+\frac{7}{8}\right) = \frac{1}{2}\times\frac{15}{16} = \frac{15}{32}$$



(b) We have $\frac{-4}{5} < \frac{7}{10}$

One rational number between $\frac{-4}{5}$ and $\frac{7}{10}$ is

$$\frac{1}{2}\left(\frac{-4}{5}+\frac{7}{10}\right) = \frac{1}{2}\left(\frac{-8+7}{10}\right) = -\frac{1}{20}.$$

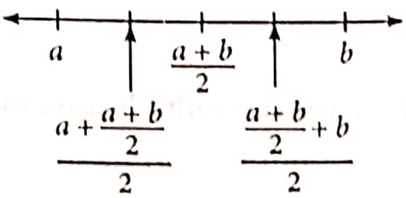


Case 2. To find more than one rational number between two given rational numbers

First Method: Mid-Value Method

If you have to find very few, say 2 or 3 rational numbers, then also you can apply the mid-value method.

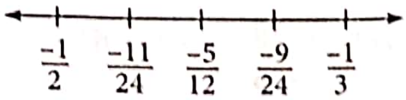
Example 6 **Find 3 rational numbers between $-\frac{1}{2}$ and $-\frac{1}{3}$.**



Solution We have, $-\frac{1}{2} < -\frac{1}{3}$.

One rational number between

$$-\frac{1}{2} \text{ and } -\frac{1}{3} = \frac{1}{2}\left[-\frac{1}{2}+\left(-\frac{1}{3}\right)\right] = \frac{1}{2}\times\frac{-5}{6} = \frac{-5}{12}$$



We may take 2nd rational number between $-\frac{1}{2}$ and $-\frac{5}{12}$.

It is equal to $\frac{1}{2}\left[-\frac{1}{2}+\left(-\frac{5}{12}\right)\right] = \frac{1}{2}\times\frac{-11}{12}=\frac{-11}{24}$.

3rd rational number may be taken between $\frac{-5}{12}$ and $-\frac{1}{3}$.

It is equal to $\frac{1}{2}\left[\left(\frac{-5}{12}\right)+\left(-\frac{1}{3}\right)\right]=\frac{1}{2}\times\frac{-9}{12}=\frac{-9}{24}$ or $\frac{-3}{8}$

The mid-value method becomes tedious and cumbersome if you have to find lots of numbers between two given rational numbers. In such cases the **gap method** as given below is easy and the most convenient.

Second Method: Gap Method

Step 1. Find the gap between the given rational numbers a and b $(a < b)$.

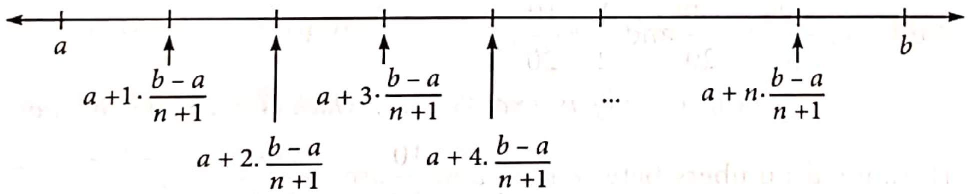
Gap = $b - a$.

Step 2. Divide this gap by $n + 1$, if you have to find n rational numbers.

Step 3. Multiply $\frac{b-a}{n+1}$ by 1, 2, 3, ... until n, and add each product to a.

Thus, the n rational numbers between rational numbers a and b are

$a+1\cdot\frac{b-a}{n+1}, a+2\cdot\frac{b-a}{n+1}, a+3\cdot\frac{b-a}{n+1}, \ldots, a+n\cdot\frac{b-a}{n+1}$



Example 7 **Find 6 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.**

Solution **Step 1.** $\frac{4}{5}>\frac{3}{5}$. Gap between $\frac{3}{5}$ and $\frac{4}{5}=\frac{4}{5}-\frac{3}{5}=\frac{1}{5}$

Step 2. To find 6 rational numbers, divide by 6 + 1 = 7.

Dividing the gap by 7, we have $\frac{1}{5}=\frac{1}{35}$.

$\therefore$ The 6 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are

$\frac{3}{5}+1\cdot\frac{1}{35}, \frac{3}{5}+2\cdot\frac{1}{35}, \frac{3}{5}+3\cdot\frac{1}{35}, \frac{3}{5}+4\cdot\frac{1}{35}, \frac{3}{5}+5\cdot\frac{3}{35}$ and $\frac{3}{5}+6\cdot\frac{1}{35}$

i.e., $\frac{22}{35}, \frac{23}{35}, \frac{24}{35}, \frac{25}{35}, \frac{26}{35}$ and $\frac{27}{35}$.

Example 8 Find 4 rational numbers between $-\frac{1}{6}$ and $\frac{1}{5}$.

Solution We have, $-\frac{1}{6}<\frac{1}{5}$. Gap between them $=\frac{1}{5}-\left(-\frac{1}{6}\right)=\frac{11}{30}$.

Since we have to find 4 rational numbers, so dividing the gap, *viz.*, $\frac{11}{30}$ by $(4+1)$, *i.e.*, 5, we have $\frac{11}{150}$.

So, the required 4 rational numbers are

$$-\frac{1}{6}+1\cdot\frac{11}{150},-\frac{1}{6}+2\cdot\frac{11}{150},-\frac{1}{6}+3\cdot\frac{11}{150} \text{ and } -\frac{1}{6}+4\cdot\frac{11}{150}.$$

$$\Rightarrow \quad \frac{-14}{150},\frac{-3}{150},\frac{8}{150} \text{ and } \frac{19}{150} \Rightarrow \frac{-7}{75},\frac{-1}{50},\frac{4}{75} \text{ and } \frac{19}{150}.$$

Third Method: Common Denominator Method

Example 9 Find 10 rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.

Solution **Method.** *Convert the denominators of the given rational numbers to a common denominator so that the difference of their numerators may be more than the number of rational numbers to be found out.*

Here, we have to find 10 rational numbers. So, change the denominators 5 and 2 to a common denominator so that the difference of numerators −2 and 1 may be more than 10.

We have, $\frac{-2}{5}=\frac{-8}{20}$ and $\frac{1}{2}=\frac{10}{20}$

Difference of 10 and −8 is 18, which is > 10 and common denominator = 20

10 rational numbers between $\frac{-8}{20}$ and $\frac{10}{20}$ are $\frac{-7}{20},\frac{-6}{20},\frac{-5}{20},\frac{-4}{20},\frac{-3}{20},\frac{-2}{20},\frac{-1}{20},0,\frac{1}{20},\frac{2}{20}$.

Self Practice 1E

Find:

1. **One rational number between**

   (a) 6 and 8 (b) −3 and 9 (c) $\frac{1}{3}$ and $\frac{1}{4}$ (d) $-\frac{1}{8}$ and $\frac{3}{16}$

2. **Two rational numbers between**

   (a) 4 and 7 (b) −1 and 6 (c) $\frac{1}{2}$ and $\frac{7}{8}$ (d) $\frac{-3}{4}$ and $\frac{5}{6}$

3. **Three rational numbers between**

   (a) −7 and −3 (b) $\frac{-2}{7}$ and $\frac{6}{7}$ (c) $\frac{3}{8}$ and $\frac{5}{12}$

4. **Five rational numbers between**

   (a) −3 and −8 (b) $\frac{3}{5}$ and $\frac{4}{5}$ (c) $-\frac{1}{6}$ and $\frac{5}{9}$

PROBLEMS BASED ON REAL LIFE SITUATIONS

Example 10 A plant that is $15\frac{7}{8}$ cm high grows x cm to a height of $19\frac{3}{8}$ cm. Write and solve an equation to find the amount of growth.

Solution Original height of the plant = $15\frac{7}{8}$ cm

New height of the plant = $19\frac{3}{8}$ cm

Height by which the plant grows = x cm

$\therefore$ Original height + x = New height

$$15\frac{7}{8}+x = 19\frac{3}{8} \Rightarrow x=19\frac{3}{8}-15\frac{7}{8}$$

$$=\frac{155}{8}-\frac{127}{8}=\frac{155-127}{8}$$

$$=\frac{\cancel{28}^{7}}{\cancel{8}_{2}}=\frac{7}{2}=3\frac{1}{2}$$

So, the amount of growth = $3\frac{1}{2}$ cm.

Example 11 **One of the first computers took $\frac{1}{5000}$ second to perform one operation. How long did it take to perform 11,000 operations?**

Solution Time taken to perform 1 operation = $\frac{1}{5000}$ sec

$\therefore$ Time taken to perform 11,000 operations = $\frac{1}{5000}\times 11000$ sec $=\frac{11}{5}$ sec $=2\frac{1}{5}$ sec.

Example 12 **$7\frac{1}{2}$ m long rope is cut into 10 equal pieces. What is the length of each piece?**

Solution Length of the rope = $7\frac{1}{2}\text{m}=\frac{15}{2}\text{m}$

Number of pieces it is cut into = 10

$\therefore$ Length of each piece = Total length ÷ Number of pieces

$$=\left(\frac{15}{2}\div 10\right)\text{m}=\frac{\cancel{15}^{3}}{2}\times\frac{1}{\cancel{10}_{2}}\text{m}=\frac{3}{4}\text{m}$$

$$=\frac{3}{\cancel{4}_{1}}\times\cancel{100}^{25}\text{ cm}=75\text{ cm}.$$

Self Practice 1F

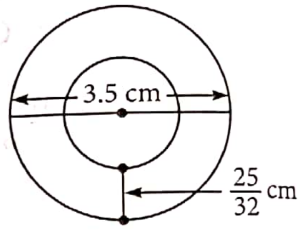
1. A group of friends hike $5\frac{3}{4}$ km, stops for lunch, and then hike another $3\frac{1}{5}$ km. How far did they hike?

2. Ms Joshi walks her dog $\frac{4}{5}$ km each day. What is the total distance that Ms Joshi walks her dog in 6 days?

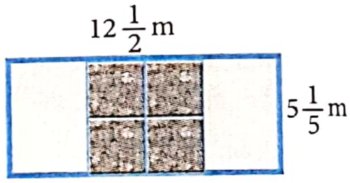
3. Priya completes $\frac{1}{20}$ of her painting each day. How much of her painting does she complete in 5 days?

4. An oxygen tank contained $219\frac{2}{3}$ L of oxygen before $32\frac{1}{3}$ L were used. If the tank can hold $245\frac{3}{8}$ L, how much space in the tank is unused?

5. A water pipe has an outside diameter of 3.5 cm and a wall thickness of $\frac{25}{32}$ cm. What is the inside diameter of the pipe?



6. Find the perimeter and area of a rectangular piece of land measuring $12\frac{1}{2}$ m by $5\frac{1}{5}$ m. How much area of the land is left unused if 4 small square flower beds of side length 1 m are made at the centre?



7. Out of a certain sum of money a boy spends $\frac{3}{5}$, and then $\frac{1}{4}$ of the remainder. He has ₹ 15 left. How much amount had he at first?

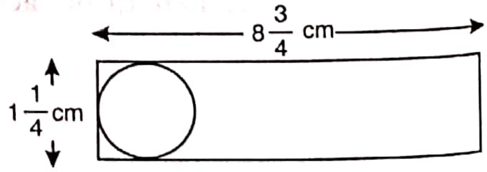
8. $10\frac{1}{2}$ tonnes of sand are to be shared between a number of builders. One of them receives $\frac{4}{7}$ of the total and the remaining sand is shared by 3 builders. How much does the fourth builder receive and how much sand does each of the other 3 builders receive?

9. A man aged 27 marries a woman aged 24, he dies at the age of 81 and she dies at the age of 91. For what fraction of his life is the man married? For what fraction of her life is the woman a widow?

10. Shalini has to cut out circles of diameter $1\frac{1}{4}$ cm from an aluminium strip of dimensions $8\frac{3}{4}$ cm by $1\frac{1}{4}$ cm. How many full circles can Shalini cut? Also, calculate the wastage of the aluminium strip. [*NCERT Exemplar*]

[**Hint:** Diameter of one circle = Breadth of the rectangular strip = $1\frac{1}{4}$ cm = $\frac{5}{4}$ cm

Length of the strip = $8\frac{3}{4}$ cm = $\frac{35}{4}$ cm



∴ Number of full circles which can be cut from the strip

$$= \frac{35}{4} \div \frac{5}{4} = \frac{35}{4} \times \frac{4}{5} = 7.$$

Wastage = Area of the strip – Total area of the 7 circles.]

Mental Maths

1. Add: $\frac{11}{12}+\frac{5}{-12}$

2. Find the difference: $\frac{-7}{9}-\left(\frac{-5}{9}\right)$

3. Additive inverse of $\frac{3}{-5}$ is ______.

4. Multiplicative inverse of $\frac{-7}{9}$ is ______.

5. $\frac{1}{3}+\frac{1}{8}=$ ______

6. $\frac{-4}{45}-\frac{11}{45}=$ ______

7. $-9\div\frac{1}{3}=$ ______

8. $\frac{-3}{4}\cdot\left(\frac{-2}{9}\right)=$ ______

9. Find one rational number between $\frac{1}{2}$ and 1. ______

10. Write a rational number equivalent to $\frac{-7}{8}$ with denominator 24. ______

Multiple Choice Questions (MCQs)

1. A number of the form $\frac{p}{q}$ is said to be a rational number, if
   (a) p, q are integers.
   (b) p, q are integers and $q \neq 0$.
   (c) p, q are integers and $p \neq 0$.
   (d) p, q are integers and $p \neq 0$ and also $q \neq 0$.

2. Which number is not equivalent to $\frac{-15}{21}$?
   (a) $\frac{10}{14}$ (b) $\frac{-5}{7}$ (c) $\frac{-45}{63}$ (d) $\frac{-75}{105}$

3. Which number is greater than $\frac{-3}{4}$?
   (a) -1 (b) $\frac{-5}{4}$ (c) $\frac{-1}{2}$ (d) -2

4. Evaluate: $\frac{-15}{16}+\left(\frac{-9}{16}\right)$.
   (a) $\frac{3}{8}$ (b) $\frac{-3}{8}$ (c) $-1\frac{1}{2}$ (d) $1\frac{1}{2}$

5. Sonam filled a fish tank with $8\frac{5}{12}$ litres of water. Shruti added $6\frac{11}{12}$ more litres of water. How many litres of water were in the tank?
   (a) $14\frac{1}{2}$ L (b) $15\frac{1}{3}$ L (c) $15\frac{5}{12}$ L (d) $15\frac{2}{3}$ L

6. What is the product of $-5\frac{1}{3}$ and $3\frac{3}{4}$?

(a) -20 (b) $-15\frac{1}{4}$ (c) $15\frac{1}{4}$ (d) 20

7. Which of the following properties does the numerical expression $\frac{8}{15}+\frac{(-11)}{30}=\frac{1}{6}$ show for rational numbers?

(a) Closure (b) Commutativity (c) Additive identity (d) Associativity

8. The multiplicative inverse of $-2\frac{3}{7}$ is

(a) $\frac{17}{7}$ (b) $\frac{7}{17}$ (c) $-\frac{7}{17}$ (d) $-\frac{17}{7}$

9. Between any two rational numbers

(a) there exists only one rational number
(b) there exist only two rational numbers
(c) there exist many rational numbers
(d) there exist infinitely many rational numbers

10. Evaluate: $\left|\frac{-5}{7}\right| \div \left|\frac{5}{7}\right|$.

(a) -1 (b) 1 (c) $\frac{-25}{49}$ (d) $\frac{49}{25}$

11. Out of the rational numbers, $\frac{-4}{7}, \frac{-5}{13}, \frac{-6}{11}, \frac{-3}{5}$ and $\frac{-2}{3}$, which one is the greatest number?

(a) $\frac{-4}{7}$ (b) $\frac{-3}{5}$ (c) $\frac{-5}{13}$ (d) $\frac{-2}{3}$

12. $\left|-8\frac{1}{3}\right| \div \left(-10\frac{5}{6}\right) \div (-2) =$

(a) $\frac{-5}{6}$ (b) $\frac{-2}{3}$ (c) $\frac{5}{13}$ (d) $\frac{11}{13}$

13. If the rational number $\frac{a}{b}$ is positive, then which of the following must be true?

(a) $a > 0$ (b) $b > 0$ (c) $ab > 0$ (d) $a + b > 0$

14. A man reads $\frac{3}{8}$ of a book on a day and $\frac{4}{5}$ of the remainder on the second day. If the number of pages still unread are 40, then how many pages did the book contain?

(a) 300 (b) 500 (c) 320 (d) 350

15. $\frac{4}{7}$ of a pole is in the mud. When $\frac{1}{3}$ of it is pulled out, 250 cm of the pole is still in the mud. Find the full length of the pole.

(a) 1000 cm (b) 1100 cm (c) 950 cm (d) 1050 cm

Chapter Test

Concept Review

1. **Fill in the blanks.**

   (a) Any number that can be written in the form $\frac{n}{d}$, where n and d are integers and $d \neq 0$, is called a ____________ number.

   (b) The product of a rational number and its multiplicative inverse is ____________.

   (c) Between $\frac{-15}{27}$ and $\frac{-20}{27}$, the greater number is ____________.

   (d) A rational number in simplest form and equivalent to $\frac{20}{45}$ is ____________.

   (e) There are ____________ rational numbers between any two rational numbers.

2. **Answer true (T) or false (F).**

   (a) The multiplicative inverse of -1 is 1.

   (b) The operation of subtraction is commutative over rational numbers.

   (c) $-\frac{2}{5}$ lies between -1 and $\frac{1}{5}$.

   (d) If $\frac{c}{d}$ is the additive inverse of $\frac{a}{b}$, then $\frac{a}{b}+\frac{c}{d}=0$.

   (e) There are 999 rational numbers between 0 and 1000.

3. **State the property shown by the following numerical statements.**

   (a) $\frac{-5}{11}\times\frac{-7}{9}=\frac{-7}{9}\times\frac{-5}{11}$

   (b) $\left(\frac{2}{3}+\frac{7}{8}\right)+\left(-\frac{1}{2}\right)=\frac{2}{3}+\left[\frac{7}{8}+\left(-\frac{1}{2}\right)\right]$

   (c) $\frac{-6}{19}+0=0+\left(\frac{-6}{19}\right)=\frac{-6}{19}$

4. Using suitable re-arrangement, find the sum $\frac{5}{9}+\left(\frac{-3}{11}\right)+\frac{7}{9}+\left(\frac{-7}{11}\right)$.

5. Verify the property $x(y+z)=xy+xz$, by taking $x=-\frac{1}{2}$, $y=\frac{3}{4}$ and $z=\frac{1}{4}$. What name is given to this property?

6. Find 3 rational numbers between $\frac{-3}{2}$ and $\frac{5}{2}$.

7. If a jar contains $\frac{3}{4}$ L of juice and each cup can hold $\frac{1}{8}$ L of juice, how many cups can be filled?

(a) $\frac{3}{32}$ cup (b) 6 cups (c) $\frac{3}{4}$ cup (d) 8 cups

8. Which value of x makes the equation $\frac{3}{5}x = -\frac{39}{25}$ true?

(a) $-1\frac{2}{5}$ (b) $-2\frac{3}{5}$ (c) $3\frac{2}{5}$ (d) $-2\frac{1}{5}$

9. How many pieces of $13\frac{1}{5}$ cm length can be cut from a 330 cm long rod?

(a) 25 (b) 28 (c) 21 (d) 35

10. If the fractions $\frac{-2}{5}, \frac{-7}{25}, \frac{1}{10}, \frac{-19}{4}, \frac{-29}{51}$ are arranged in ascending order of their values, then which one will be the first?

(a) $\frac{-29}{7}$ (b) $\frac{-2}{5}$ (c) $\frac{1}{10}$ (d) $\frac{-19}{4}$